

# Appendix (for online publication)

## A Proofs for the simple model

### A.1 Optimal choice of recourse under symmetric information

I derive the optimal choice of recourse in the absence of asymmetric information friction.

**No binding SGC.** In this case the quantity of sold securities  $s_i x_i$  is demand determined, i.e., is independent on resources of issuing lender, and therefore also independent on sale price  $q_i(r_i^{EG})$  and the recourse choice, in equilibrium. The optimal choice of explicit recourse solves:

$$\frac{\partial w_i}{\partial r_i^{EG}} = \frac{\partial[r^h(n_i + (q_i - 1)) - \max\{r^{EG} - r^h, 0\}](1 + \tau)]s_i x_i}{\partial r_i^{EG}} = \left(r^h \frac{\partial q_i}{\partial r_i^{EG}} - 1 - \tau\right) s_i x_i = 0.$$

Using the market clearing condition  $q_i = r^{EG} q^h / r^h$ , the above condition collapses to  $q^h = 1 + \tau$ . This implies that only when  $q^h \geq 1 + \tau > 1$  lenders provide explicit recourse. Similarly the optimal choice of implicit recourse conditional on being credible (i.e. satisfying the non-default condition 11) solves:

$$\frac{\partial w_i}{\partial r_i^{IG}} = \frac{\partial[r^h(n_i + (q_i - 1)) - \max\{\chi_i r^{IG} - r^h, 0\}]s_i x_i}{\partial r_i^{IG}} = \left(r^h \frac{\partial q_i}{\partial r_i^{IG}} - 1\right) s_i x_i = 0,$$

which collapses to  $q^h = 1$ . But when  $q^h = 1$ , securitization is not profitable, which implies that no credible implicit recourse exceeding a loan return can be provided. As a result the equilibrium level of implicit recourse is given by the maximum recourse satisfying condition (11) with equality. Therefore, when the SGC is not binding (i.e., when  $q^h = 1$ ), neither explicit nor implicit recourse exceeding the loan return is provided.

**Binding SGC.** In this case the quantity of sold securities is supply determined  $s_i x_i = \theta x_i$ , i.e., it depends on the resources of the issuing lender, and therefore also on sale price  $q_i(r_i^{EG})$  and recourse. The optimal choice of explicit recourse then solves:

$$\frac{\partial w_i}{\partial r_i^{EG}} = \frac{\partial}{\partial r_i^{EG}} \frac{r^h(1 - \theta) - \theta \max\{r_i^{EG} - r^h, 0\}(1 + \tau)}{1 - \theta q_i} n_i = 0,$$

which collapses to

$$q^h = (1 + \tau) / (1 + \theta\tau). \tag{A.1}$$

In equilibrium the recourse provision lowers the price of high-quality securities without recourse  $q^h = \frac{r^h}{r^{EG}} q^G = \frac{r^h}{r^{EG}} \frac{1 - \pi\mu}{\theta}$ , which brings the price to the equilibrium level given by (A.1). Similarly, the optimal choice of implicit recourse conditional on being credible (i.e., satisfying 11) solves:

$$\frac{\partial w_i}{\partial r_i^{IG}} = \frac{\partial}{\partial r_i^{IG}} \frac{r^h(1 - \theta) - \theta \max\{\chi_i r^{IG} - r^h, 0\}}{1 - \theta q_i} n_i = 0,$$

which again collapses to  $q^h = 1$ . But since under  $q^h = 1$  securitization is not profitable, the equilibrium level of implicit recourse is again given by the maximum recourse satisfying the non-default condition (11) with equality.

## A.2 Proof of Claim 2

**Full characterization of the equilibrium.** Suppose condition (15) in Proposition 2 is satisfied and neither explicit nor credible explicit recourse are available. Then the equilibrium is characterized by one of the following cases:<sup>1</sup>

1. Separating equilibrium (Case *H*): Only lenders with high-quality loans invest (takes place if  $q^l < \frac{\pi\mu}{1-\theta} < 1$ )
2. Pooling equilibrium: lenders with both high- and low-quality loans invest:
  - Case *M*: Lenders with low-quality loans invest with probability  $\phi$  (takes place if  $\frac{\pi\mu}{1-\theta} \leq q^l \leq \frac{\pi}{1-\theta}$ );
  - Case *B*: All lenders with low-quality loans invest and securitize loan returns (takes place if  $q^l > \frac{\pi}{1-\theta}$ )

**Case *H*** takes place when lenders with low-quality securities refrain from investing, i.e., condition (12) is satisfied, which under asymmetric information takes the form of (17). Using the market clearing condition  $r^h/q^h = r^l/q^l$  and substituting the equilibrium price  $q^h$  from eq. (14), I can rewrite (17) as:

$$q^l < \frac{1 - \theta q^h}{1 - \theta} = \frac{\pi\mu}{1 - \theta} < 1. \quad (\text{A.2})$$

or equivalently as minimum requirement on the return dispersion:

$$\frac{r^h}{r^l} > q^h \frac{1 - \theta}{1 - \theta q^h} = \frac{(1 - \theta)(1 - \pi\mu)}{\pi\mu\theta}.$$

**Case *M*** takes place when lenders with low-quality securities are mixing, i.e., they are indifferent about buying securities in the market and investing:  $r^h/q^h = (1 - \theta)r^l/(1 - \theta q)$ , where price  $q$  reflects the average security quality bringing average return  $\mu r^h + \phi(1 - \mu)r^l$ . The indifference condition can be rewritten as follows:  $q^l = (1 - \theta q)/(1 - \theta) = \pi(\mu + (1 - \mu)\phi)/(1 - \theta)$ .

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<sup>1</sup>Note that prices  $q^l, q^l$  under asymmetric information are market prices of securities of known quality with returns  $r^h$  and  $r^l$ , respectively. I drop subscript  $i$  because the equilibrium is symmetrical, meaning that lenders with the same loans choose the same controls.

**Case B** takes place when both lenders with high- and low- quality loans invest and securitize to the maximum capacity, i.e., when

$$q^l > \frac{1 - \theta q}{1 - \theta} = \frac{\pi}{1 - \theta},$$

where the second equality follows from  $q = \frac{1 - \pi}{\theta}$ , which can be obtained by the combination of aggregate investment equation  $\sum_i x_i = \frac{\pi N}{1 - \theta q}$  with the goods market clearing condition  $N = \sum_i x_i$ .

### A.3 Proof of Claim 4 and Proposition 3

Suppose that  $q^l < 1$ , then the equation (22) can be rearranged to the expression for the maximum recourse that lenders are willing to provide to separate:

$$\frac{r^{EG}}{r^h} = \frac{1 - \pi\mu + \theta\tau}{\theta(1 + \tau)} > 1. \quad (\text{A.3})$$

Substituting the expression for the recourse A.3 into the non-mimicking condition 20 gives:

$$\frac{r^h}{r^l} > \frac{(1 - \pi\mu)(1 + \tau)}{(1 - \pi\mu)(1 + \tau) - \tau\pi\mu(1 - \pi\mu - \theta)}. \quad (\text{A.4})$$

For  $\tau > 1$  the RHS exceeds 1 since  $1 - \pi\mu - \theta > 0$  by (15) in Proposition 2. Condition A.4 is a necessary condition, since in case  $q^l > 1$ , maximum recourse will be lower which would increase the RHS of A.4.

It remains to prove that condition (20) is satisfied for larger subset of parameters than condition (17). This is the case because while the denominator on the RHS is the same in both conditions, the numerator on the RHS is smaller in condition (20) if  $r^{EG} > r^l$ . Moreover, if  $r^{EG} > r^h$ , then the left-hand side (LHS) of (20) is larger than (17), given that  $q^h |_{r^{EG}=0} = (1 - \pi\mu)/\theta > q^h |_{r^{EG}>r^h} = r^h q / r^G = (1 - \pi\mu)r^h / (\theta r^{EG})$ .

### A.4 Proof of Claim 5

I first define all pooling and separating PBE and then refine them using the intuitive criterion. Finally, I derive conditions for the existence of the separating equilibrium. For simplicity, I assume that the explicit recourse is not available,  $r^{EG} = 0$ .

**Pooling PBE.** In order to identify pooling equilibria, it is useful to establish several points. First, analogously to Claim 1, there is no pooling equilibrium in cases where all investing lenders choose a level of recourse lower than the maximum recourse on which lenders selling

low-quality securities would not default  $r^{IG} \leq r_{l,cred,p}^{IG}$  when  $r_{l,cred,p}^{IG} > r^h$ .<sup>2</sup> Because, due to competition, lenders with both high- and low-quality loans increase implicit recourse to  $r_{l,cred,p}^{IG}$ . The beliefs of saving lenders do not affect the security price since both types honor the implicit recourse on this interval.

Second, when recourse exceeds  $r_{l,cred,p}^{IG}$ , lenders selling low-quality securities default on the recourse and the security owner receives  $r^l < r_{l,cred,p}^{IG}$ . This negatively affects the market price and disincentivizes increasing the recourse above  $r_{l,cred,p}^{IG}$ .

Third, there are two natural upper limits for the implicit recourse in a pooling equilibrium. The first is the maximum credible recourse level that a lender selling high-quality securities can provide:  $r_{h,cred,p}^{IG}$ .<sup>3</sup> The second limit is the level of recourse at which all lenders selling low-quality securities prefer to separate  $r_{minsep}^{IG}$ .<sup>4</sup>

The above-mentioned considerations lead to the definition of the whole set of pooling PBE, which consist of three types.

**Case B:** Lenders with high- and low-quality loans choose recourse  $r^{IG*} = r_{l,cred,p}^{IG}$ . Saving lenders' out-of-equilibrium beliefs that sustain this equilibrium can be the following:  $\varphi_{i,1} |_{r_i^{IG} > r_{l,cred,p}^{IG}} = 0$  and unrestricted for intervals  $0 < r_i^{IG} < r_{l,cred,p}^{IG}$ . In this equilibrium, no lender defaults on an implicit recourse.

**Case B':** Lenders with high and low-quality loans select  $r_i^{IG} = r^{IG*}$  such that:

$$r_{lb,p}^{IG} \leq r^{IG*} \leq \min \{ r_{minmix}^{IG}, r_{h,cred,p}^{IG} \}.$$

Saving lenders' out-of-equilibrium beliefs that sustain this equilibrium can be the following:

$\varphi_i |_{r_i^{IG} > r^{IG*}} = 0$ , and  $\varphi_i |_{r_i^{IG} < r^{IG*}} \leq \mu$ .<sup>5</sup>

**Case M:** Lenders with high-quality loans and a fraction  $\phi$  of lenders with low-quality

<sup>2</sup>At the recourse  $r_{l,cred,p}^{IG}$  in the pooling equilibrium, lenders selling low-quality securities are indifferent between defaulting and honoring the recourse:  $\nu^{ND} w_i |_{r_i^{IG} = r_{l,cred,p}^{IG}} = \nu^D w_i |_{r_i^{IG} = r_{l,cred,p}^{IG}} \quad \forall i \in \mathcal{L}$ .

<sup>3</sup>At recourse  $r_{h,cred,p}^{IG}$  in the pooling equilibrium, lenders selling high-quality securities are indifferent between defaulting and honoring the recourse:  $\nu^{ND} w_i |_{r_i^{IG} = r_{h,cred,p}^{IG}} = \nu^D w_i |_{r_i^{IG} = r_{h,cred,p}^{IG}} \quad \forall i \in \mathcal{H}_1$ .

<sup>4</sup>At recourse  $r_{minsep}^{IG}$ , only lenders with high-quality loans invest. All lenders with low-quality loans choose not to invest and are marginally indifferent between buying securities in the market and mimicking lenders with high-quality loans:  $\nu^{ND} w_i |_{x_i=0} = \nu^D w_i |_{x_i>0, \varphi_{i,1}=1, r_i^{IG} = r_{minsep}^{IG}} \quad \forall i \in \mathcal{L}_1$ .

<sup>5</sup> $r_{minmix}^{IG}$  is the recourse in an equilibrium, where all lenders with high- and low-quality loans invest, securitize and provide this recourse, but lenders with low-quality loans are marginally indifferent between this strategy and buying securities in the market.  $r_{lb,p}^{IG}$  is the recourse in an equilibrium, where all lenders with high- and low-quality loans invest, securitize and provide this recourse, but lenders with high-quality loans are marginally indifferent between this strategy and deviating to provision of  $r_{l,cred,p}^{IG}$  even if this implies  $\varphi_i = 0$ . There is no pooling equilibrium with  $r_{l,cred,p}^{IG} < r^{IG} < r_{lb,p}^{IG}$ , since both types have incentives to decrease implicit recourse to  $r_j^{IG} = r_{l,cred,p}^{IG}$ . This is because the negative price effect of equilibrium defaults on recourse by sellers of low-quality loans, together with additional costs of the higher implicit recourse, outweighs the positive price effect of the higher implicit recourse.

loans invest and choose  $r_i^{IG} = r^{IG*}$  such that:

$$\min \{r_{\text{minmix}}^{IG}, r_{h,\text{cred},p}^{IG}\} < r^{IG*} < \min \{r_{\text{minsep}}^{IG}, r_{h,\text{cred},p}^{IG}\}.$$

Saving lenders' out-of-equilibrium beliefs that sustain this equilibrium can be the same as in the Case  $B_1^A$ .

**Separating PBE.** There is potentially a continuum of separating PBE, where lenders with low-quality loans save and buy securities from lenders with high-quality loans. Lenders with high-quality loans invest, securitize, and provide implicit recourse  $r^{IG*} \in [r_{\text{minsep}}^{IG}, r_{h,\text{cred},s}^{IG}]$ , where  $r_{h,\text{cred},s}^{IG}$  is the maximum implicit recourse that lenders selling high-quality securities can credibly provide in a separating equilibrium.

Saving lenders' out-of-equilibrium beliefs that sustain this equilibrium are for instance the following:  $\varphi_i |_{r_i^{IG} > r^{IG*}} = 0$  and unrestricted for  $r_i^{IG} < r^{IG*}$ .

**Application of the intuitive criterion:** If any separating equilibrium exists, then all pooling equilibria are dominated, and therefore, fail the intuitive criterion (Cho and Kreps, 1987). Due to competition, the intuitive criterion selects the separating equilibrium with maximum credible implicit recourse  $r^{IG*} = r_{h,\text{cred},s}^{IG}$ , which is given by (25). See Figure A.1 for illustration. In cases where no separating PBE exists, application of the intuitive criterion does not affect potential multiplicity of pooling equilibria. However, for a subset of parameters there is a unique equilibrium (see Figure A.2).

The **unique separating equilibrium** takes place when condition (12) is satisfied, which takes the form (23). Since lenders issuing high-quality securities are indifferent about recourse, then any issuer of low-quality securities would prefer to default as:

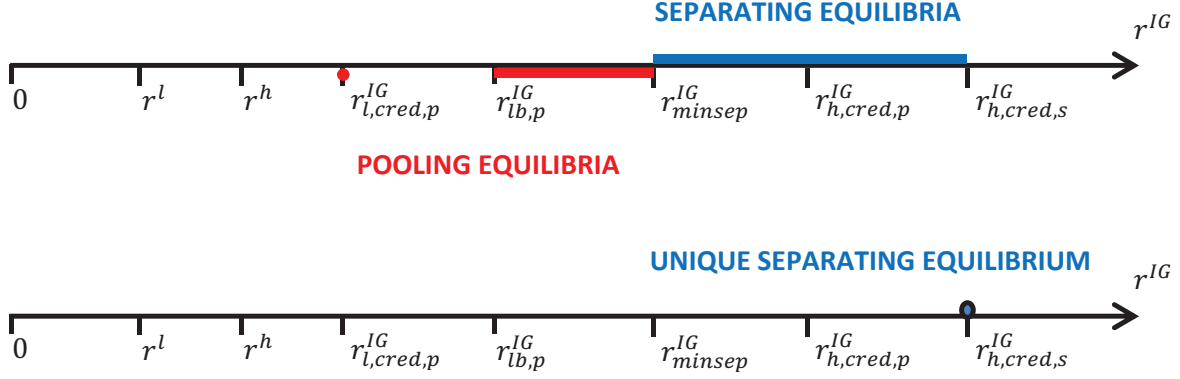
$$\frac{r^l - \frac{\theta}{1-\theta}(r^{IG} - r^l)}{q^S} \nu^{ND} < \frac{r^l}{q^S} \nu^D. \quad (\text{A.5})$$

Using this observation and combining conditions (23) and (25) gives (24).

A **unique pooling equilibrium** takes place when only Case  $B$  is an equilibrium, i.e. when

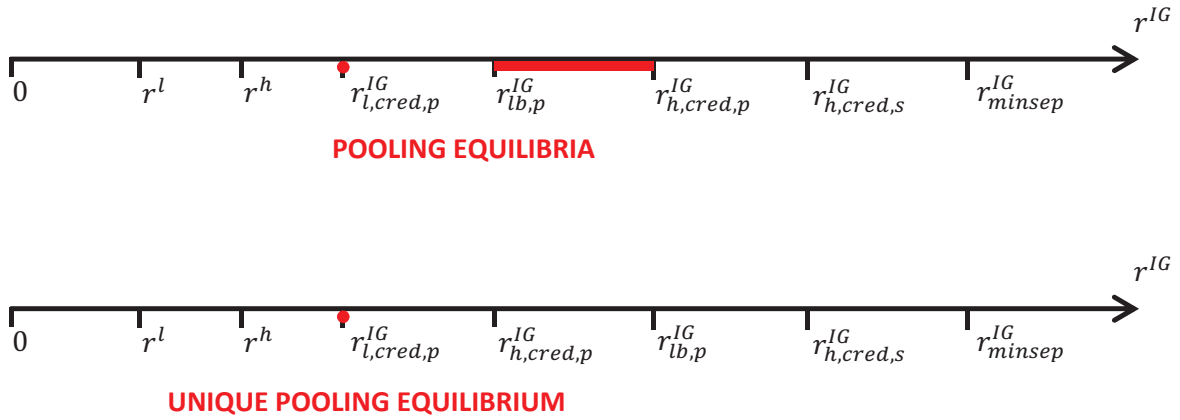
1.  $r_{lb,p}^{IG}$  satisfying  $r_{lb,p}^{IG} > r_{l,\text{cred},p}^{IG}$  exceeds the maximum credible level of recourse that can be provided by lenders with high-quality loans:  $r_{lb,p}^{IG} > r_{h,\text{cred},p}^{IG}$ ; or when
2. there is no recourse satisfying all properties for  $r_{lb,p}^{IG}$ . Recall that  $r_{lb,p}^{IG} > r_{l,\text{cred},p}^{IG}$  is the equilibrium recourse level at which lenders with high-quality loans are indifferent between providing such recourse or deviating from the equilibrium by providing a recourse  $r_{l,\text{cred},p}^{IG}$ . As I show below for sufficiently small rates of occurrence of high-quality

Figure A.1. Intuitive criterion selects a unique separating equilibrium



Note: On the upper axis I highlight the implicit recourse level for a continuum of separating and pooling PBE in blue and red, respectively. The intuitive criterion selects a unique separating equilibrium with  $r_i^{IG} = r_{h,cred,s}^{IG} \forall i \in \mathcal{H}_1$  (on the lower axis).

Figure A.2. Separating equilibrium does not exist



Note: The minimum recourse level needed for separation  $r_{minsep}^{IG}$  exceeds the maximum level of recourse that can be credibly provided without default  $r_{h,cred,s}^{IG}$ . Therefore, no separating equilibrium exists and the intuitive criterion does not eliminate any PBE. There remains a multiplicity of pooling equilibria (upper panel) or a unique pooling equilibrium with  $r^{IG} = r_{l,cred,p}^{IG}$  (lower panel). The latter takes place because  $r_{lb,p}^{IG}$  exceeds the maximum credible recourse provided in a pooling equilibrium  $r_{h,cred,p}^{IG}$ .

loans ( $\mu$ ), there are no  $r_{lb,p}^{IG} > r_{l,cred,p}^{IG}$  that would satisfy this indifference condition.

Lenders with high-quality loans prefer to deviate to  $r_{l,cred,p}^{IG}$  if

$$\begin{aligned} V_i |_{r^{IG}=r_{l,cred,p}^{IG}+\Delta} &< V_i |_{r^{IG}=r_{l,cred,p}^{IG}} \\ \frac{r^h - \theta (r_{l,cred,p}^{IG} + \Delta)}{1 - \theta \left( \mu \frac{r_{l,cred,p}^{IG} + \Delta}{r^h} + (1 - \mu) \frac{r^l}{r^h} \right) q^h} &< \frac{r^h - \theta r_{l,cred,p}^{IG}}{1 - \theta \frac{r_{l,cred,p}^{IG}}{r^h} q^h} \end{aligned} \quad (\text{A.6})$$

Condition A.6 holds for all parameters satisfying the following, more restrictive, condition:

$$\frac{r^h - \theta (r_{l,cred,p}^{IG} + \Delta)}{1 - \theta \left( \mu \frac{r_{l,cred,p}^{IG} + \Delta}{r^h} + (1 - \mu) \frac{r^l}{r^h} \right) q^h} < \frac{r^h - \theta r_{l,cred,p}^{IG}}{1 - \theta \left( \mu \frac{r_{l,cred,p}^{IG}}{r^h} + (1 - \mu) \frac{r^l}{r^h} \right) q^h},$$

which can be simplified to

$$\begin{aligned} \Delta \theta \left( 1 - \theta q^h \left( \mu \frac{r_{l,cred,p}^{IG}}{r^h} + (1 - \mu) \frac{r^l}{r^h} \right) \right) &> \Delta \theta \mu q^h \left( 1 - \theta \frac{r_{l,cred,p}^{IG}}{r^h} \right) \\ \mu &< \frac{1 - \theta q^l}{q^h - q^l \theta}. \end{aligned}$$

Intuitively, for low occurrence of high-quality loans  $\mu$ , the negative price effect of defaults by lenders with low-quality loans dominates the positive effects of a higher recourse.

## A.5 Endogenizing the “skin in the game”

Consider a simple moral hazard problem, which endogenizes the existence of the SGC. Lenders can divert lending funds, but still issue and sell securities backed by returns from non-existing loans. This can be observed only in period 2. But before selling securities to buyers, the issuing lender can be asked to prove that it has spent a fraction of the intended lending from its own resources. The lending retained by the issuing lender is verifiable. Therefore, this moral hazard problem can be eliminated if security buyers require the issuing lenders to retain a sufficiently large “skin in the game,” which would satisfy the incentive compatible constraint (ICC):

$$V |_{diverting \text{ lending funds}} \leq V |_{lending \text{ properly}}.$$

When a lender diverts funds, its return on endowment is  $R |_{diverting \text{ investment funds}} = \left( \frac{\theta q_i}{1 - \theta_i} \right)^{\iota_i}$ , because it has to spend  $1 - \theta_i$  per unit of lending from its own resources, which cannot be recovered, and it receives  $\theta_i q_i$  per unit of lending.  $\iota_i$  is the number of times the lender

reuses the returns from this operation to repeat this fund diversion scheme. Since there is no restriction of sequential security issuance (a practice using the funds from the security sale to cover the “skin in the game” fraction of a new investment within the same period), then for an infinitely small lender  $\iota_i$  is unbounded. As a result, the ICC always fails unless  $\theta_i q_i \leq (1 - \theta_i)$ , or equivalently unless

$$\theta_i \leq \frac{1}{q_i + 1}. \quad (\text{A.7})$$

Intuitively, the higher the sale price of loans  $q_i$ , the larger “skin in the game”  $(1 - \theta_i)$  is required to prevent the moral hazard problem. Substituting the price in (A.7) from (14), we find that  $\theta_i \leq \pi\mu$ . Therefore, when fraction of endowment owned by lenders with high-quality lending opportunity is less than the fraction of endowment owned by other lenders,  $\pi\mu < 1/2$ , then SGC binds and the price of loans is equal to  $q_i = \frac{1-\pi\mu}{\pi\mu}$ .

## B Derivation of lenders’ policy functions in the full model

I characterize the recursive equilibrium (I drop the time subscripts and use  $\prime$  to denote next-period variables).

**Definition B.1.** *A recursive competitive perfect Bayesian equilibrium consists of prices and price functions  $\{q^h(\bar{\mathbf{S}}_t), q^l(\bar{\mathbf{S}}_t), \{q_j^P(\bar{\mathbf{S}}_t, r_j^{EG'}, r_j^{IG'}, \varphi_j)\}, \{q_j^S(\bar{\mathbf{S}}_t, \varphi_j^S)\}\} \forall j$ , gross profits per unit of capital  $\{r^h(\bar{\mathbf{S}}_t), r^l(\bar{\mathbf{S}}_t)\}$ , individual decision rules  $\{c(\bar{s}_{i,t}; \bar{\mathbf{S}}_t), x(\bar{s}_{i,t}; \bar{\mathbf{S}}_t), h^O(\bar{s}_{i,t}; \bar{\mathbf{S}}_t), l^O(\bar{s}_{i,t}; \bar{\mathbf{S}}_t), r^{EG'}(\bar{s}_{i,t}; \bar{\mathbf{S}}_t), r^{IG'}(\bar{s}_{i,t}; \bar{\mathbf{S}}_t), \{a_j^P(\bar{s}_{i,t}, r_j^{EG'}, r_j^{IG'}, \varphi_j; \bar{\mathbf{S}}_t)\}, \{a_j^S(\bar{s}_{i,t}, \varphi_j^S; \bar{\mathbf{S}}_t)\}, \chi(\bar{s}_{i,t}; \bar{\mathbf{S}}_t)\} \forall j$ , value functions  $\{V^{ND}(\bar{s}_{i,t}; \bar{\mathbf{S}}_t), V^D(\bar{s}_{i,t}; \bar{\mathbf{S}}_t)\}$ , and the law of motion for  $\bar{\mathbf{S}}_t = \{K_t, \omega_t, A_t, \Sigma_t\}$  such that:*

- (i) *individual decision rules and value functions solve lenders’ problems and are sequentially rational given their beliefs taking price functions, gross profits per unit of capital, and law of motion for  $\bar{\mathbf{S}}_t$  as given;*
- (ii) *both security and goods markets clear;*
- (iii) *lenders update their beliefs using the Bayes’ rule on the equilibrium path based on other lenders observable controls  $\{\varphi_j(b_{j,t})\} \forall j$ ; and*
- (iv) *the law of motion for  $\bar{\mathbf{S}}$  is consistent with the individual lender’s decisions.*

The set of individual state variables early in the period  $t$  is  $\bar{\mathbf{s}}_{i,t}^e = \{x_{i,t-1}, \{a_{i,j,t-1}^P\}, \{a_{i,j,t-1}^S\}, h_{i,t-1}^O, l_{i,t}^O, r_{i,t}^{EG}, r_{i,t}^{IG}, \sigma_{i,t-1}\} \forall j$  and late in the period is  $\bar{\mathbf{s}}_{i,t} = \{\bar{\mathbf{s}}_{i,t}^e, \kappa_{i,t}, \chi_{i,t}\}$ .



From the FOCs, I can obtain the following Euler equations in cases where the SGC is binding for all investing lenders:

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{\hat{r}_{j,t+1} + \lambda q_{j,t+1}^S}{q_{j,t}^P} \right] = 1 \quad \forall i \in \mathcal{S}_t, \forall j \in \mathcal{I}_t, \quad (\text{B.1})$$

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{j,t+1} + \lambda (\varphi_{j,t}^S q_{t+1}^h + (1 - \varphi_{j,t}^S) q_{t+1}^l)}{q_{j,t}^S} \right] = 1 \quad \forall i \in \mathcal{S}_t, \forall j \in \mathcal{I}_{t-1}, \quad (\text{B.2})$$

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} \right] = 1 \quad \forall i \in \mathcal{S}_t, \quad (\text{B.3})$$

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right] = 1 \quad \forall i \in \mathcal{S}_t, \quad (\text{B.4})$$

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{(1 - \theta) (r_{t+1}^h + \lambda q_{i,t+1}^S) - \theta (g_{i,t+1}^T - y_{i,t+1})}{1 - \theta q_{i,t}^P} \right] = 1 \quad \forall i \in \mathcal{H}_t \cap \mathcal{I}_t, \quad (\text{B.5})$$

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{(1 - \theta) (r_{t+1}^l + \lambda q_{i,t+1}^S) - \theta (g_{i,t+1}^T - y_{i,t+1})}{1 - \theta q_{i,t}^P} \right] = 1 \quad \forall i \in \mathcal{L}_t \cap \mathcal{I}_t, \quad (\text{B.6})$$

where  $\varphi_{j,t}^S$  is the belief of lenders in period  $t$  that the security issued by lender  $j$  in period  $t - 1$  is of high quality. Due to the logarithmic utility function, all lenders consume a  $(1 - \beta)$  fraction of their wealth:

$$c_{i,t} = (1 - \beta) w_{i,t} \quad \forall i, t. \quad (\text{B.7})$$

Under the binding SGC, investing lenders ( $\mathcal{I}_t$ ) invest all of the unconsumed part of their wealth into new loans and sell the maximum fraction of investment  $\theta$  to saving lenders:<sup>6</sup>

$$a_{i,i,t}^P = \frac{\beta w_{i,t}}{\frac{(1 - \theta q_{i,t}^P)}{(1 - \theta)}} \quad \forall i \in \mathcal{I}_t,$$

Saving lenders  $\mathcal{S}_t$  are, in equilibrium, indifferent about investing in different securities. All of them try to diversify their investment, so I guess and verify that, in equilibrium, all will allocate the same fraction of wealth into different securities:

$$\begin{aligned} q_{j,t}^P a_{i,j,t}^P &= \zeta_j^{hP} \beta w_{i,t} \quad \forall i \in \mathcal{S}_t \quad \forall j \in \mathcal{H}_t \cap \mathcal{I}_t, \\ q_{j,t}^P a_{i,j,t}^P &= \zeta_j^{lP} \beta w_{i,t} \quad \forall i \in \mathcal{S}_t \quad \forall j \in \mathcal{L}_t \cap \mathcal{I}_t, \\ q_{j,t}^S a_{i,j,t}^S &= \zeta_j^{hS} \beta w_{i,t} \quad \forall i \in \mathcal{S}_t \quad \forall j \in \mathcal{H}_{t-1} \cap \mathcal{I}_{t-1}, \\ q_{j,t}^S a_{i,j,t}^S &= \zeta_j^{lS} \beta w_{i,t} \quad \forall i \in \mathcal{S}_t \quad \forall j \in \mathcal{L}_{t-1} \cap \mathcal{I}_{t-1}, \\ q_t^h h_{i,t}^O &= \zeta^{hO} \beta w_{i,t} \quad \forall i \in \mathcal{S}_t, \\ q_t^l l_{i,t}^O &= \zeta^{lO} \beta w_{i,t} \quad \forall i \in \mathcal{S}_t. \end{aligned}$$

---

<sup>6</sup>This applies in cases of separating equilibrium as well as in cases of pooling equilibrium where all lenders with low-quality loans invest to the full capacity.

Wealth fractions sum up to one:  $\sum_{j \in \mathcal{H}_t \cap \mathcal{I}_t} \zeta_j^{hP} + \sum_{j \in \mathcal{H}_t \cap \mathcal{I}_t} \zeta_j^{lP} + \sum_{j \in \mathcal{H}_{t-1} \cap \mathcal{I}_{t-1}} \zeta_j^{hS} + \sum_{j \in \mathcal{L}_{t-1} \cap \mathcal{I}_{t-1}} \zeta_j^{lS} + \zeta^{hO} + \zeta^{lO} = 1$ . Since lenders' decision rules are either independent of or linear in wealth, we do not have to keep track of the wealth distribution  $\Sigma_t$ .

The consumption of lenders in the following period depends on the return from their investment:

$$\begin{aligned} c_{i,t+1} &= (1 - \beta) \left[ \sum_{j \in \mathcal{I}_t} a_{i,j,t}^P (\hat{r}_{j,t+1} + \lambda q_{j,t+1}^S) + \sum_{j \in \mathcal{H}_{t-1} \cap \mathcal{I}_{t-1}} a_{i,j,t}^S (r_{t+1}^h + \lambda q_{t+1}^h) \right. \\ &\quad \left. + \sum_{j \in \mathcal{L}_{t-1} \cap \mathcal{I}_{t-1}} a_{i,j,t}^S (r_{t+1}^l + \lambda q_{t+1}^l) + h_{i,t}^O (r_{t+1}^h + \lambda q_{t+1}^h) + l_{i,t}^O (r_{t+1}^l + \lambda q_{t+1}^l) \right] \forall i \in \mathcal{S}_t, \\ c_{i,t+1} &= (1 - \beta) a_{i,i,t} \left( r_{t+1}^h + \lambda q_{i,t+1}^S - \frac{\theta}{(1 - \theta)} (g_{i,t+1}^T - y_{i,t+1}) \right) \forall i \in \mathcal{H}_t \cap \mathcal{I}_t, \\ c_{i,t+1} &= (1 - \beta) a_{i,i,t} \left( r_{t+1}^l + \lambda q_{i,t+1}^S - \frac{\theta}{(1 - \theta)} (g_{i,t+1}^T - y_{i,t+1}) \right) \forall i \in \mathcal{L}_t \cap \mathcal{I}_t. \end{aligned}$$

Using these guesses in (B.5) and (B.6), it is clear the latter conditions always hold.

The stochastic discount factor  $\beta c_{i,t} / c_{i,t+1}$  in the remaining Euler equations (B.1), (B.2), (B.3) and (B.4) can be rewritten as:  $1 / \Xi_{t+1} \equiv \beta c_{i,t} / c_{i,t+1}$ , where

$$\begin{aligned} \Xi_{t+1} &= \sum_{j \in \mathcal{H}_t \cap \mathcal{I}_t} \zeta_i^{hP} \frac{\hat{r}_{j,t+1} + \lambda q_{j,t+1}^S}{q_{j,t}^P} + \sum_{j \in \mathcal{L}_t \cap \mathcal{I}_t} \zeta_j^{lP} \frac{\hat{r}_{j,t+1} + \lambda q_{j,t+1}^S}{q_{j,t}^P} \\ &\quad + \sum_{j \in \mathcal{H}_{t-1} \cap \mathcal{I}_{t-1}} \zeta_j^{hS} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_{j,t}^S} + \sum_{j \in \mathcal{L}_{t-1} \cap \mathcal{I}_{t-1}} \zeta_j^{lS} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_{j,t}^S} \\ &\quad + \zeta^{hO} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^{lO} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}. \end{aligned}$$

## C Full-model solution in the deterministic steady state

In this appendix, I re-derive analytically selected propositions from Section 3 for the deterministic steady state of the full model.

### C.1 Cases without binding SGC: first-best

If the SGC is not binding, only lenders with high-quality lending opportunities lend and because of competition, the security price is  $q^h = 1$ . Lenders with low-quality loans do not mimic lenders with high-quality loans because the separating condition

$$V_{i,t} |_{mimicking} < V_{i,t} |_{buying\ securities} \quad \forall i \in \mathcal{L}_t, \quad (\text{C.1})$$

which collapses to  $A^h > A^l$  is always satisfied. Due to logarithmic utility, lenders always consume a  $1 - \beta$  fraction of their wealth:  $c = (1 - \beta) h (r^h + \lambda)$ , which aggregates to

$$C = (1 - \beta) H (r^h + \lambda).$$

Combining the good market clearing condition  $X = Y - C = Hr^h - C$  with the law of motion for capital  $X = (1 - \lambda) H$ , I obtain:

$$\begin{aligned} Hr^h - C &= (1 - \lambda) H \\ Hr^h - (1 - \beta) H (r^h + \lambda) &= (1 - \lambda) H, \\ r^h + \lambda &= \frac{1}{\beta}. \end{aligned}$$

## C.2 Cases with binding SGC

The SGC is binding when it restricts investment of lenders with high-quality loans, i.e.  $a_{i,i,t}^P = (1 - \theta) x_{i,t} \forall i \in \mathcal{H}_t$ . Their budget constraints (27) become

$$c_{i,t} + (1 - \theta q_t^h) x_{i,t} = w_{i,t} \forall i \in \mathcal{H}_t. \quad (\text{C.2})$$

Substituting for  $c_{i,t}$  from (B.7) in (C.2), I get their investment function  $x_{i,t}^h = \beta w_{i,t} / (1 - \theta q_t^h) \forall i \in \mathcal{H}_t$ , which aggregates to  $X_t^H = \pi \mu \beta W_t / (1 - \theta q_t^h)$ . Combining it with the aggregate version of (28),  $W_t = H_t (r^h + \lambda q^h)$ , we can rewrite it in the steady state as

$$(1 - \lambda) (1 - \theta q^h) = \pi \mu \beta (r^h + \lambda q^h). \quad (\text{C.3})$$

The goods market clearing condition  $Y_t = X_t + C_t$  becomes in the steady state:

$$r^h = (1 - \lambda) + (1 - \beta) (r^h + \lambda q^h). \quad (\text{C.4})$$

Combining (C.3) with (C.4), I obtain the market price:

$$q^h = \frac{(1 - \lambda) (1 - \pi \mu)}{(1 - \lambda) \theta + \pi \mu \lambda}. \quad (\text{C.5})$$

The SGC is binding only if  $q^h > 1$ , i.e., when  $(1 - \lambda) (1 - \pi \mu) > (1 - \lambda) \theta + \pi \mu \lambda$  (which is equivalent to  $(1 - \theta) > \pi \mu / (1 - \lambda)$ ). This leads to the next proposition.

**Proposition C.1.** *If the “skin in the game” is sufficiently large to satisfy*

$$(1 - \lambda) (1 - \theta) > \pi \mu, \quad (\text{C.6})$$

*then the SGC binds and the price of high-quality securities exceeds the lending costs,  $q^h > 1$ .*

I denote  $R_{i,t+1}$  as the gross return on wealth:  $R_{i,t+1} = w_{i,t+1} / w_{i,t}$ . First, assume that

explicit and credible implicit recourse are not available. Then the separating condition (C.1) can be rewritten as follows when the condition (C.6) holds:<sup>7</sup>

$$\begin{aligned} R_{i,t+1} |_{mimicking} &< R_{i,t+1} |_{buying\ securities} \quad \forall i \in \mathcal{L}_t, \\ \frac{r^l + \lambda q^l}{\frac{1-\theta q^h}{1-\theta}} &< \frac{r^h + \lambda q^h}{q^h}, \\ q^l &< \frac{1 - \theta q^h}{1 - \theta}. \end{aligned}$$

Substituting for  $q^h$  from (C.5) and using  $\frac{A^h}{q^h} = \frac{A^l}{q^l}$ , I get:

$$\frac{A^h}{A^l} > \frac{(1 - \theta) q^h}{1 - \theta q^h} = \frac{(1 - \theta) (1 - \pi\mu) (1 - \lambda)}{\pi\mu (\theta + \lambda (1 - \theta))}. \quad (\text{C.7})$$

When only the explicit recourse is available, then the separating condition (C.1) becomes:

$$\frac{r^h + \lambda q^h}{q^h} > \frac{(1 - \theta) (r^l + \lambda q^l) - \theta [\max \{r^{EG} - r^l, 0\} - \tau \max \{r^{EG} - r^h, 0\}]}{1 - \theta q}. \quad (\text{C.8})$$

The maximum level of recourse that lenders selling high-quality securities are willing to offer to separate from sellers of low-quality securities is given by the indifference between return when providing recourse and separating and return when considered as a seller of low-quality securities:

$$\frac{(1 - \theta)(r^h + \lambda q^h) - \theta(r^{EG} - r^h)(1 + \tau)}{1 - \theta q} = (r^h + \lambda q^h) \max \left\{ 1, \frac{1 - \theta}{1 - \theta q^l} \right\} \quad (\text{C.9})$$

Suppose that  $q^l < 1$ , then the above condition can be simplified to

$$\frac{r^{EG} + \lambda q^h}{r^h + \lambda q^h} = \frac{1 + \theta\tau - (1 - \theta q)}{\theta(1 + \tau)}. \quad (\text{C.10})$$

Substituting maximum recourse (C.10) into the non-mimicking condition (C.8) gives:

$$\frac{A^h}{A^l} > \frac{(1 + \tau)q(1 - \lambda\theta\zeta)}{(1 + \tau)q(1 - \lambda\theta\zeta) - \tau(1 - \theta q)(q - 1)}, \quad (\text{C.11})$$

where

$$\zeta = q^h / (r^h + \lambda q^h). \quad (\text{C.12})$$

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<sup>7</sup>Note that if no recourse is provided, all securities reveal their quality in the period following their issuance, at the latest.

Since (C.6) holds, the RHS of (C.7) exceeds one; and since  $\tau > 1$ , the RHS of (C.11) exceeds one too. Therefore, a pooling equilibrium exists for the low productivity dispersions without as well as with the provision of explicit recourse.

When only the implicit recourse is available and the intuitive criterion is applied to refine PBEs, the separating condition (C.1) can be summarized in the the following proposition.

**Proposition C.2.** *Suppose condition (C.6) holds and only implicit recourse is available. Then a separating equilibrium is possible in the deterministic steady state if and only if*

$$\frac{A^h}{A^l} > \frac{(1 - \theta P) q^h}{1 - \theta P q^h} = \frac{(1 - \theta P) (1 - \pi \mu) (1 - \lambda)}{P \pi \mu (\theta + \lambda (1 - \theta))}, \quad (\text{C.13})$$

where  $P \equiv \frac{q^P}{q^h} = \frac{r^{IG} + \lambda q^h}{r^h + \lambda q^h} > 1$  is the price premium for the equilibrium implicit recourse. This implies that the separating equilibrium is more likely in the presence of an implicit recourse, but since the RHS of (C.13) exceeds one, a pooling equilibrium exists for the low productivity dispersions which do not satisfy (C.13).

**Proof.** Since  $P > 1$  (see later in the proof), when comparing separating conditions (C.7) and (C.13), it is straightforward to show that condition (C.13) is satisfied for a larger parameter subspace.

I proceed by deriving the equilibrium solution and the condition (C.13) from the separating condition (C.1) and showing that the RHS of the inequality (C.13) exceeds one and is independent on  $A^h$  and  $A^l$ .

The steady-state conditions for the separating PBE, which satisfies the intuitive criterion, are as follows:

$$\text{Investment function:} \quad (1 - \lambda) (1 - \theta q^P) = \pi \mu \beta (r^h + \lambda q^h), \quad (\text{C.14})$$

$$\text{Goods market clearing:} \quad r^h = (1 - \lambda) + (1 - \beta) (r^h + \lambda q^h), \quad (\text{C.15})$$

$$\text{Security market clearing:} \quad \frac{\hat{r} + \lambda q^h}{q^P} = \frac{r^h + \lambda q^h}{q^h}, \quad (\text{C.16})$$

$$\text{Binding non-default cond.:} \quad V^{ND} (w' |_{\chi'=1}) = V^D (w' |_{\chi'=0}) \quad \forall i \in \mathcal{H}. \quad (\text{C.17})$$

Using the following property given by the logarithmic utility function:

$$V(w) = \log((1 - \beta) w) + \beta \log((1 - \beta) \beta R w) + \beta^2 \log((1 - \beta) \beta^2 R^2 w) + \dots = \frac{1}{1 - \beta} \log(w) + V(1),$$

I write lenders' value functions in the following way:

$$\begin{aligned}
V^D(w' |_{x'=0}) &= V^D(1) + \frac{1}{1-\beta} \log \left( \beta \frac{(1-\theta)(r^h + \lambda q^h)}{(1-\theta q^P)} w \right) \\
V^{ND}(w' |_{x'=1}) &= V^{ND}(1) + \frac{1}{1-\beta} \log \left( \beta \frac{(1-\theta) \left( r^h + \lambda q^h - \frac{\theta}{1-\theta} (r^{IG} - r^h) \right)}{(1-\theta q^P)} w \right).
\end{aligned}$$

Value functions with unitary wealth can be obtained as follows:

$$\begin{aligned}
V^{ND}(1) &= \log(1-\beta) + \beta \left( \pi \mu V^{ND}(\beta R^{h,ND}) + \pi(1-\mu) V^{ND}(\beta R^l) + (1-\pi) V^{ND}(\beta R^z) \right) \\
&= \log(1-\beta) + \beta \left( \frac{\pi \mu \log(\beta R^{h,ND})}{1-\beta} + \pi(1-\mu) \frac{\log(\beta R^l)}{1-\beta} + (1-\pi) \frac{\log(\beta R^z)}{1-\beta} + V^{ND}(1) \right) \\
&= \frac{\log(1-\beta)}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2} \left( \pi \mu \log(R^{h,ND}) + \pi(1-\mu) \log(R^l) + (1-\pi) \log(R^z) \right). \\
V^D(1) &= \frac{\log(1-\beta)}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2} \left( \pi \mu \log(R^{h,D}) + \pi(1-\mu) \log(R^l) + (1-\pi) \log(R^z) \right),
\end{aligned}$$

where  $R^{h,ND}$ ,  $R^{h,D}$  are one-period returns on wealth for lenders with high-quality loans that have never defaulted on implicit recourse and for those that have defaulted on implicit recourse, respectively.  $R^l$  and  $R^z$  are one period returns on wealth for lenders with low-quality loans and no lending opportunity, respectively. Substituting the above equations into the non-default condition (C.17) and canceling the terms equal for both value functions, I obtain:

$$\begin{aligned}
&\log \left( \beta (1-\theta) \left( r^h + \lambda q^h - \frac{\theta}{1-\theta} (r^{IG} - r^h) \right) \right) + \frac{\beta \pi \mu}{1-\beta} \log(R^{h,ND}) \\
&= \log(\beta (1-\theta) (r^h + \lambda q^h)) + \frac{\beta \pi \mu}{1-\beta} \log(R^{h,D}),
\end{aligned}$$

where the LHS shows the utility from consumption when wealth is reduced by repayment of the implicit recourse and from the future discounted benefit of having a good reputation. The RHS, then, shows higher immediate utility from savings on the implicit recourse, but the future utility is lower, since the lender can no longer issue and sell new loans. This equation can further be simplified using (C.16) and substituting for the returns:

$$\begin{aligned}
-\log \left( \frac{r^h + \lambda q^h - \theta (r^{IG} + \lambda q^h)}{(1-\theta)(r^h + \lambda q^h)} \right) &= \frac{\beta \pi \mu}{1-\beta} \log \left( \frac{R^{h,ND}}{R^{h,D}} \right) \\
&= \frac{\beta \pi \mu}{1-\beta} \log \left( \frac{(1-\theta) \left( r^h + \lambda q^h - \frac{\theta}{1-\theta} (r^{IG} - r^h) \right)}{(1-\theta q^P)} \frac{1}{(r^h + \lambda q^h)} \right) \\
&= \frac{\beta \pi \mu}{1-\beta} \log \left( \frac{r^h + \lambda q^h - \theta (r^{IG} + \lambda q^h)}{r^h + \lambda q^h - \theta q^h (r^{IG} + \lambda q^h)} \right).
\end{aligned}$$

Finally, this non-default condition can be expressed as follows:

$$\log\left(\frac{1-\theta}{1-\theta P}\right) = \frac{\beta\pi\mu}{1-\beta} \log\left(\frac{1-\theta P}{1-\theta P q^h}\right). \quad (\text{C.18})$$

The LHS is the ratio of immediate utility from defaulting to immediate utility from honoring the implicit recourse. The RHS is the ratio of the discounted sum of future utilities from honoring the recourse to the discounted sum of future utilities from defaulting on recourse. Note that the argument of the logarithm on the RHS corresponds to  $R^{h,ND}/R^{h,D}$  and exceeds one only if  $q^h > 1$ . Indeed only then is securitization profitable. Equation (C.18) also implies that only if  $q^h > 1$ , is the RHS positive and as a result only then  $P > 1$ . This can be intuitively interpreted that only when securitization is profitable, lenders suffer from punishment and they can provide a credible implicit recourse.

The steady-state condition (C.18) together with the following condition (obtained by combining C.14 and C.15)

$$q^h = \frac{(1-\lambda)(1-\pi\mu)}{(1-\lambda)\theta P + \pi\mu\lambda} \quad (\text{C.19})$$

determine the solution to  $q^h$  and  $P$ , which depends only on the time preference ( $\beta$ ), depreciation ( $\lambda$ ) and financial frictions parameters ( $\pi, \mu, \theta$ ). Therefore,  $q^h$  and  $P$  do not depend on the productivity levels  $A^h$  and  $A^l$ , which is the first step of the proof. Note also that if condition (C.6) holds, then condition (C.19) implies that  $q^h > 1$ .

The second step is to derive (C.13) from (C.1). Using similar transformations as with condition (C.17), I rewrite the separation condition (C.1):<sup>8</sup>

$$\begin{aligned} \log\left(\frac{\beta(1-\theta)(r^l + \lambda q^l)}{(1-\theta q^P)}\right) + \frac{\beta\pi\mu}{1-\beta} \log(R^{h,D}) &< \log\left(\beta\frac{(r^h + \lambda q^h)}{q^h}\right) + \beta\pi\mu \log(R^{h,ND}) \\ \log\left(\frac{(r^l + \lambda q^l)(1-\theta)}{(1-\theta q^P)} \frac{q^h}{(r^h + \lambda q^h)}\right) &< \frac{\beta\pi\mu}{1-\beta} \log\left(\frac{R^{h,ND}}{R^{h,D}}\right) \\ \log\left(\frac{(1-\theta)q^l}{(1-\theta P q^h)}\right) &< \frac{\beta\pi\mu}{1-\beta} \log\left(\frac{1-\theta P}{1-\theta P q^h}\right). \end{aligned}$$

I substitute the RHS of the above condition using (C.18) to get:

$$\begin{aligned} \log\left(\frac{(1-\theta)q^l}{(1-\theta P q^h)}\right) &< \log\left(\frac{1-\theta}{1-\theta P}\right) \\ \frac{A^h}{A^l} &> \frac{(1-\theta P)q^h}{1-\theta P q^h} = \frac{(1-\theta P)(1-\pi\mu)(1-\lambda)}{P\pi\mu(\theta + \lambda(1-\theta))}. \end{aligned} \quad (\text{C.20})$$

The equality in (C.20) is obtained by substituting for  $q^h$  from (C.19). The RHS of (C.20)

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<sup>8</sup>Recall that in the separating equilibrium, selected by the intuitive criterion, mimicking lenders with low-quality loans choose to default on implicit recourse, since  $r^{IG} > r_{l,cred,s}^{IG}$ .

exceeds one, because when condition (C.6) holds,  $q^h > 1$  and  $P > 1$  (see earlier in the proof).

## D Numerical solution of the fully stochastic dynamic model

To capture the effect of switching between a separating and a pooling equilibrium, I use global numerical approximation methods for the model solution. In particular, I find the numerical approximation for endogenous variables on a grid of state variables  $\bar{S} = (A, K, \omega)$  by iteration. The Matlab codes for the numerical solution are available online.<sup>9</sup>

The model solution can be reduced to finding the solution for the set of three non-predetermined endogenous variables  $q^h(\bar{S})$ ,  $q^l(\bar{S})$  and  $V^{diff}(\bar{S}) \equiv V^{ND}(\bar{S}) - V^D(\bar{S})$ , which I denote  $\bar{\Gamma}(\bar{S}) = \{q^h, q^l, V^{diff}\} | \bar{S}$ . Expectations about their next-period values determine the current level of all endogenous variables. Once I know  $\bar{\Gamma}(\bar{S})$ , I can find the remaining endogenous variables, including the law of motion for the endogenous state variables  $K$  and  $\omega$ . Therefore, all equilibrium conditions can be written as  $E(\bar{\Gamma}, \bar{\Gamma}', \bar{S}'(\bar{\Gamma})) | \bar{S} = 0$ .

I use the following algorithm to find the numerical approximation to the model solution.

**Initiation:** I construct a three-dimensional grid  $\mathbb{S}$  of state variables as all possible combinations of  $\bar{A}, \bar{K}$  and  $\bar{\omega}$ , which are vectors of selected nodes for state variables. Since aggregate productivity takes only two values, I choose  $\bar{A} = \{A^H, A^L\}$ . Vector  $\bar{K}$  consists of  $n$  equidistant values for  $K$  with the median being the steady-state value of  $K$ . Vector  $\bar{\omega}$  consists of  $n$  equidistant values from the interval of possible values for  $\omega$  in equilibrium, i.e., from  $[\mu, 1]$ .

I make an initial guess for the value function difference on the grid:  $\bar{V}_0^{diff}(\mathbb{S}) = \{V_0^{diff}\} | \bar{S} \in \mathbb{S}$ . I choose the stopping criterion  $\varepsilon > 0$  and set the value function iteration counter to zero,  $l = 0$ .

**Step 1:** I do nested iteration to find out the values of  $\{q^h, q^l\} | \bar{S} \in \mathbb{S}$  for the particular guess of the value function difference  $V_l^{diff}$ .<sup>10</sup> I make an initial guess for the remaining non-predetermined endogenous variables of interest on the grid  $\{\bar{q}_0^h(\mathbb{S}), \bar{q}_0^l(\mathbb{S})\} = \{q_0^h, q_0^l\} | \bar{S} \in \mathbb{S}$  and set the price iteration counter to zero,  $k = 0$ .

- **Step 1a:** For all combinations of state variables on the grid  $\forall \bar{S} \in \mathbb{S}$ , I compute  $V_{l+1}^{diff}$  and  $q_{k+1}^h, q_{k+1}^l$ , which satisfy<sup>11</sup>

$$E(\bar{\Gamma}_{l+1, k+1}, \bar{\Gamma}'_{l, k}, \bar{S}'(\bar{\Gamma}_{l, k})) | \bar{S} = 0.$$

<sup>9</sup> [www.martin-kuncl.info/files/Securitization\\_Numerical\\_solution.zip](http://www.martin-kuncl.info/files/Securitization_Numerical_solution.zip)

<sup>10</sup>The subscript for  $V_l^{diff}$  denotes the value function iteration number.

<sup>11</sup>The first subscript of  $\bar{\Gamma}$  denotes the number of the value function iteration and the second subscript, the number of the price function iteration:  $\bar{\Gamma}_{l+1, k+1} = \{q_{k+1}^h, q_{k+1}^l, V_{l+1}^{diff}\}$ .



Note that  $\bar{\Gamma}'_{l,k}$  is a function of  $\bar{S}'(\bar{\Gamma}_{l,k})$ , which might be in between the grid points. In this case, I use linear interpolation on the values of the neighboring grid points on the state space.

- **Step 1b:** If the difference between the values of the two subsequent iterations for prices is smaller than the stopping criterion, i.e., if

$$\| \bar{q}_{k+1}^h(\mathbb{S}) - \bar{q}_k^h(\mathbb{S}) \| + \| \bar{q}_{k+1}^l(\mathbb{S}) - \bar{q}_k^l(\mathbb{S}) \| < \varepsilon,$$

then I move to Step 2; otherwise, I go back to Step 1a with the price iteration counter  $k$  increased by one.

**Step 2:** If the difference between the values of the two subsequent iterations for the difference in value functions is smaller than the stopping criterion, i.e., if  $\| \bar{V}_{l+1}^{diff}(\mathbb{S}) - \bar{V}_l^{diff}(\mathbb{S}) \| < \varepsilon$ , then I move to Step 3; otherwise, I go back to Step 1 with the iteration counter  $l$  increased by one.

**Step 3:** I declare  $\bar{\Gamma}_{l,k}(\mathbb{S})$  the final approximate solution and compute the remaining endogenous variables in the model.

## E Explicit recourse in the dynamic model

In the main text, I assume that the parameter capturing regulatory costs of explicit recourse,  $\tau$ , is so high that the explicit recourse is not provided in equilibrium. In this appendix, I show that lower regulatory costs,  $\tau$ , increase the explicit recourse provision, which in turn decreases the issuance of low quality loans, and reduces the output volatility and negative skewness of output growth.

Figure E.1 shows the effect of  $\tau$  on the share of lenders with low-quality lending opportunity that issue loans,  $\phi$ , and on the share of explicit recourse. The latter is defined as a fraction of explicit recourse in total recourse, where size of recourse is computed as the excess of guaranteed return over the lowest possible return realization of the underlying loans in the next period:

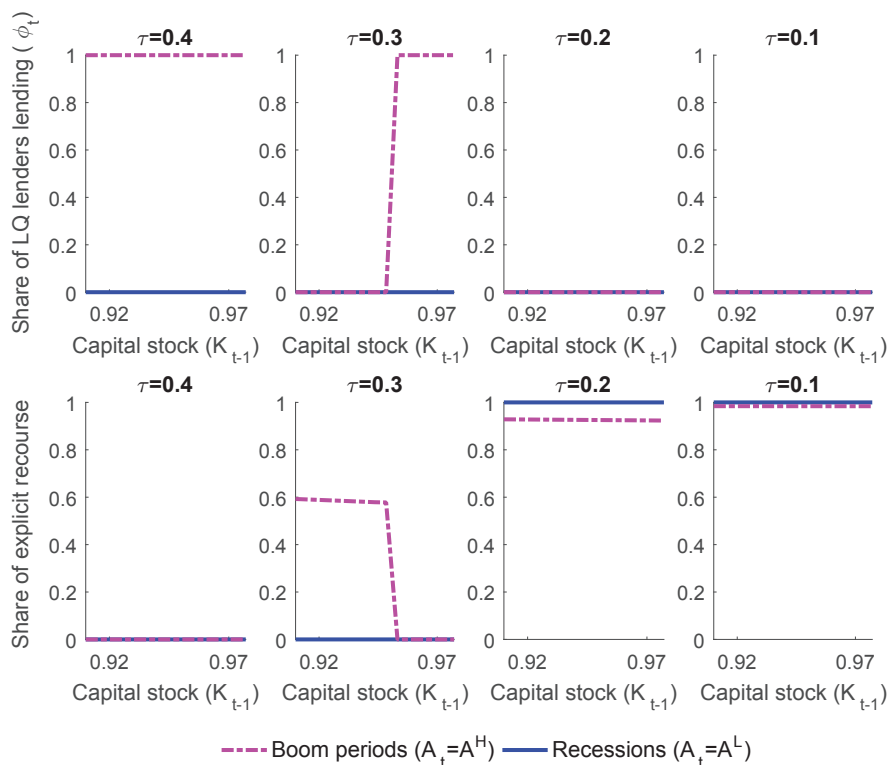
$$\text{Share of explicit recourse} = \frac{\max(r_{t+1}^{EG} - E \min r_{t+1}, 0)}{\max(\max\{r_{t+1}^{IG}, r_{t+1}^{EG}\} - E \min r_{t+1}, 0)}. \quad (\text{E.1})$$

Lowest possible return realization is  $E \min r_{t+1} = r_{t+1}^h(A_{t+1} = A^L)$  in a separating equilibrium, and  $E \min r_{t+1} = r_{t+1}^l(A_{t+1} = A^L)$  in a pooling equilibrium.

For high regulatory costs,  $\tau \geq 0.33$ , no explicit recourse exceeding the underlying loan return is provided in equilibrium, and therefore the economy behaves the same as in Section

5. That is, economy switches between a separating equilibrium in recessions and a pooling equilibrium in boom periods. When we start to reduce  $\tau$ , first explicit recourse is used only to signal quality. Therefore, for  $\tau = 0.3$ , we can see that explicit recourse is provided only in boom periods for low levels of capital (relatively higher loan returns), when its provision makes the equilibrium separating,  $\phi = 0$ . Finally, if we continue to lower  $\tau$  further, more explicit recourse will be provided but not to signal security quality, because the equilibrium is already separating. Instead, due to competition, issuing lenders increase the recourse on sold securities because recourse becomes more affordable. This is illustrated on panels for  $\tau = 0.2$  and  $\tau = 0.1$ . Higher provision of explicit recourse reduces the profits from securitization, which in turn lowers the size of implicit recourse that can be credibly pledged.

Figure E.1. Cheaper explicit recourse results in more efficient signaling



Note: Top row of panels show  $\phi_t$  and the bottom row of panels show the share of explicit recourse, defined in (E.1). Both are plotted as a function of  $K_{t-1}$  and for different levels of  $\tau$ .

Table E.1 shows the effect of lower  $\tau$  on the standard deviation of output and the skewness of output growth. For the interval  $\tau \geq 0.33$ , both output statistics remain unchanged. When we lower  $\tau$  below 0.33, explicit recourse is used to signal security quality, which lowers the proportion of low-quality loans in the economy. Since return on low-quality loans is more volatile, fewer low-quality loans reduce the output volatility. Lower variation in asymmetric

information (variation in  $\phi$ ) over the business cycle reduces the negative skewness of output growth, because the economy features lower intensity of the build-up of stock of low-quality loans in boom and cleansing of balance sheets in bust. Finally, note that once the equilibrium is separating for all states that are visited over the cycle, further reduction in  $\tau$  has no effects on aggregate output, but only on the redistribution of returns among lenders. Higher recourse reduces profits from securitization, and therefore reduces the difference in return between issuing lenders and buying lenders.

Table E.1. Lower  $\tau$  reduces volatility of output and negative skewness of output growth<sup>a</sup>

Regulatory costs ( $\tau$ )	0.4	0.33	0.3	0.2	0.1
Standard deviation	2.09	2.09	1.8	1.68	1.68
Skewness	-1.56	-1.56	-0.96	0.12	0.12

<sup>a</sup> Output statistics obtained from 20,000 simulated observations: Standard deviation computed for percentage output deviation from mean and skewness is of the first-differenced output.

## F Appendix to the empirical analysis

The analysis was performed in Stata and the respective codes are available on [http://www.martin-kuncl.info/files/Securitization\\_Empirical\\_codes.zip](http://www.martin-kuncl.info/files/Securitization_Empirical_codes.zip).

**Data summary statistics.** Table F.1 shows the summary statistics for the variables used from the PDS database before and after Winsorizing and controlling for trend and persistence in the first-stage regression.

**First-stage regression.** To avoid potential spurious regression problem I clear *Overcollat* from potential trend and persistence in a first-stage regression, which I run on the level of individual deals (for every  $i$ ) to account for heterogeneity:

$$Overcollat_{i,t} = \alpha_i^{FS} + \beta_i^{FS} Overcollat_{i,t-1} + \gamma_i^{FS} Deal\ age_{i,t} + \epsilon_{i,t} \forall i,$$

and I use residuals from the regression equation  $Overcollat_{i,t}^D \equiv \epsilon_{i,t} \forall i$  in the second-stage (eq. 2).

**Robustness checks.** First, I run the same regression on the sub-sample excluding the late-2000s crisis and post-crisis period. Results in Table F.2 on the signaling of recourse and weaker signaling for loans issued in boom are comparable to those in Table 1 for the

UK. Statistical significance for other countries disappears possibly due to lower number of observations. This suggests that the crisis episode does not determine the results for the UK.

Second, I Winsorize both delinquency rates and the overcollateralization rates at the 2.5%-level to account for potential data errors and limit the effect of potential outliers. The regression results reported in Table F.4 are qualitatively similar to the results in Table 1 except for Ireland whose parameters of interest retain the sign, but are lower and not significant from zero.

Third, I show that the results are not driven by controlling for trend and persistence of the *Overcollat<sup>D</sup>* variable. I use the actual *Overcollat* directly in the main regression of interest (eq. 2) and obtain results reported in Table F.5, which are qualitatively similar to those in Table 1.

Finally, I show in Table F.3 that when the regression is run on the subset of deals issued in the boom stage of the business cycle, I cannot find support for the signaling relationship in the UK and the signaling effect is much smaller and less statistically significant in Ireland. This result is in line with the hypothesis of weaker or non-existent signaling for deals issued in the boom stage.

Table F.1. Data summary statistics

	Definition	Transf.	Country	(1) Obs.	(2) Mean	(3) Std. Dev.	(4) Min	(5) Max	
<i>DelinqRate</i>	Percentage ratio of receivables 90 or more days past due to the original pool balance	-	UK	4,373	4.55	4.54	0	26.65	
			IR	1,466	2.67	4.72	0	29.41	
			SP	5,933	0.53	0.93	0	13.41	
			NL	4,202	0.55	1.45	0	23.29	
			IT	1,978	1.00	0.93	0	9.57	
		Winsorized	UK	4,373	4.29	3.89	0	12.02	
			IR	1,466	2.31	3.46	0	12.02	
			SP	5,933	0.53	0.92	0	12.02	
			NL	4,202	0.54	1.34	0	12.02	
			IT	1,978	1.00	0.93	0	9.57	
<i>Overcollat</i>	Percentage difference between the principal value of collateral asset and the principal value of deal tranches normalized by the original collateral principal value	-	UK	4,564	0.18	5.08	-99.71	52.47	
			IR	1,645.00	-5.39	57.61	-646.25	85.32	
			SP	6,009	-0.63	2.16	-32.04	34.21	
			NL	4,418.00	-0.94	6.25	-117.48	90.64	
			IT	2,491.00	-2.04	6.03	-38.35	36.52	
		detrended	UK	4,360	0.00	0.75	-21.66	21.64	
			IR	1,576	-0.00	2.86	-81.16	54.40	
			SP	5,781	0.00	1.07	-26.63	24.45	
			NL	4,208	-0.00	1.72	-31.12	72.77	
			IT	2,346	-0.00	3.27	-22.52	28.93	
			Winsorized & detrended	UK	4,360	-0.00	0.38	-2.72	1.93
				IR	1,576	-0.04	0.55	-2.72	1.93
				SP	5,781	-0.00	0.48	-2.72	1.93
				NL	4,208	-0.03	0.60	-2.72	1.93
			IT	2,346	-0.07	1.38	-2.72	1.93	

Table F.2. Regression on pre-crisis subsample<sup>a</sup>

	(1)	(2)	(3)	(4)	(5)	(6)
Countries	all 5 countries	UK	IR	SP	NL	IT
$DelinqRate_{i,t-1}$	0.834*** (0.022)	0.766*** (0.027)	0.754*** (0.028)	0.411* (0.213)	0.681*** (0.033)	0.602*** (0.089)
$Overcoll_{i,t-1}^D$	-0.000 (0.003)	-0.109** (0.054)	0.033 (0.047)	-0.001 (0.001)	-0.003 (0.002)	0.004 (0.004)
$Overcoll_{i,t-1}^D \times$ $D_i^{origin\ in\ boom}$	-0.001 (0.003)	0.122** (0.056)	-0.035 (0.047)	-0.001 (0.003)	0.002 (0.003)	-0.009 (0.006)
$Deal\ age_{i,t}$	-0.039 (0.033)	-0.157 (0.129)	-0.065 (0.060)	0.029 (0.033)	-0.014 (0.017)	0.139 (0.086)
$Overcoll_{i,t-1}^D \times$ $D_{i,t}^{boom}$	0.001 (0.003)	0.027 (0.061)	0.005 (0.004)	0.002 (0.002)	0.000 (0.003)	0.013 (0.012)
$Output\ gap_{i,t}$	4.641*** (1.420)					
Observations	4,700	1,010	485	1,584	1,266	355
R-squared	0.683	0.762	0.693	0.254	0.579	0.480
Number of deals	411	115	38	118	105	35

<sup>a</sup> Time period excludes the late-2000s financial crisis and the following period. For the whole sample of countries the time period is 1998Q2-2007Q2; 2000Q2-2007Q2 for the UK; 1998Q3-2007Q2 for Spain; and 1998Q2-2007Q2 for the Netherlands.

Table F.3. Regression on loans issued in boom

	(1)	(2)	(3)	(4)	(5)	(6)
Countries	all 5 countries	UK	IR	SP	NL	IT
$DelinqRate_{i,t-1}$	0.915*** (0.013)	0.861*** (0.010)	0.998*** (0.017)	0.826*** (0.018)	0.896*** (0.022)	0.655*** (0.033)
$Overcoll_{i,t-1}^D$	0.004 (0.005)	0.006 (0.010)	-0.004* (0.002)	0.029 (0.018)	0.040 (0.032)	-0.003 (0.005)
$Deal\ age_{i,t}$	-0.029 (0.024)	-0.177*** (0.039)	-0.203 (0.146)	0.012 (0.044)	-0.004 (0.026)	0.129* (0.070)
$Overcoll_{i,t-1}^D \times$ $D_{i,t}^{boom}$	-0.002 (0.005)	0.039 (0.049)	0.001 (0.003)	-0.008 (0.021)	-0.041 (0.033)	0.005 (0.008)
$Output\ gap_{i,t}$	-3.066** (1.301)					
Observations	8,012	2,459	752	2,420	1,695	686
R-squared	0.885	0.895	0.984	0.758	0.855	0.607
Number of deals	411	112	34	111	102	52

Table F.4. Regression on Winsorized data

Countries	(1) all 5 countries	(2) UK	(3) IR	(4) SP	(5) NL	(6) IT
$DelinqRate_{i,t-1}^W$	0.907*** (0.008)	0.872*** (0.009)	0.958*** (0.016)	0.831*** (0.023)	0.885*** (0.014)	0.650*** (0.033)
$Overcoll_{i,t-1}^{D,W}$	-0.003 (0.007)	-0.081** (0.039)	-0.079 (0.077)	-0.002 (0.014)	-0.002 (0.005)	0.005 (0.011)
$Overcoll_{i,t-1}^{D,W} \times D_i^{origin\ in\ boom}$	0.014 (0.010)	0.132*** (0.047)	0.062 (0.080)	0.074** (0.035)	-0.008 (0.006)	-0.019 (0.013)
$Deal\ age_{i,t}$	-0.016 (0.015)	-0.127*** (0.027)	-0.068** (0.027)	-0.019 (0.026)	-0.001 (0.018)	0.134*** (0.047)
$Overcoll_{i,t-1}^{D,W} \times D_{i,t}^{boom}$	0.002 (0.008)	0.020 (0.054)	0.024 (0.029)	0.002 (0.025)	0.002 (0.005)	0.030** (0.013)
$Output\ gap_{i,t}$	-0.847 (0.647)					
Observations	16,303	3,949	1,346	5,486	3,791	1,731
R-squared	0.873	0.899	0.965	0.738	0.919	0.558
Number of deals	788	190	60	227	195	116

Table F.5. Without first-stage regression

Countries	(1) all 5 countries	(2) UK	(3) IR	(4) SP	(5) NL	(6) IT
$DelinqRate_{i,t-1}$	0.892*** (0.010)	0.835*** (0.012)	0.975*** (0.020)	0.807*** (0.022)	0.895*** (0.012)	0.663*** (0.032)
$Overcoll_{i,t-1}$	-0.006 (0.005)	-0.046*** (0.011)	-0.511*** (0.112)	-0.001 (0.003)	0.009 (0.006)	0.004 (0.003)
$Overcoll_{i,t-1} \times D_i^{origin\ in\ boom}$	0.013* (0.007)	0.046** (0.021)	0.510*** (0.111)	0.042*** (0.015)	0.032 (0.030)	-0.003 (0.003)
$Deal\ age_{i,t}$	-0.040** (0.018)	-0.150*** (0.028)	-0.300** (0.133)	-0.017 (0.026)	0.001 (0.018)	0.141*** (0.047)
$Overcoll_{i,t-1} \times D_{i,t}^{boom}$	-0.001* (0.000)	0.004 (0.013)	-0.000 (0.000)	0.001 (0.006)	-0.020 (0.017)	0.005 (0.003)
$Output\ gap_{i,t}$	-0.379 (0.878)					
Observations	17,068	4,141	1,405	5,703	3,976	1,843
R-squared	0.870	0.885	0.962	0.734	0.883	0.573
Number of deal	798	194	61	227	197	119

## References

Cho, I.-K., Kreps, D. M., 1987. Signaling games and stable equilibria. *The Quarterly Journal of Economics* 102 (2), 179–222.